

Extensions of Navier-Stokes-Euler Governing Equations of Fluid Flow to Fractional Time and Multi-fractional Space

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Abstract

This paper describes the recently developed governing equations of unsteady multi-dimensional incompressible and compressible flow in fractional time and multi-fractional space. When their fractional powers in time and in multi-fractional space are specified to unit integer values, the developed fractional equations of continuity and momentum for incompressible and compressible fluid flow reduce to the conventional Navier-Stokes equations. As such, these fractional governing equations for fluid flow may be interpreted as generalizations of the conventional Navier-Stokes equations. The derived governing equations of fluid flow in fractional differentiation framework herein are nonlocal in time and space. Therefore, they can quantify the effects of initial and boundary conditions better than the conventional Navier-Stokes equations. For the frictionless flow conditions, the corresponding fractional governing equations were also developed as a special case of the fractional governing equations of incompressible flow. When their derivative fractional powers are specified to unit integers, these equations are shown to reduce to the conventional Euler equations. Numerical simulations were also performed to investigate the merits of the proposed fractional governing equations. It is shown that the developed equations are capable of simulating anomalous sub- and super-diffusion due to their nonlocal behavior in time and space.